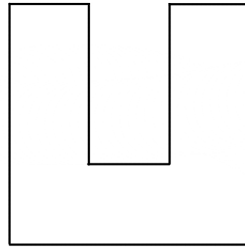


Time Limit: 15 minutes total, 2.5 minutes per submission.

Instructions: This subround consists of 6 questions, all of which have **integer answers**. You will be given all 6 questions at once (below), but you **must submit a previously unsubmitted problem every 2.5 minutes**. If a single problem is submitted multiple times, **only the first submission will be counted and all other submissions will be deemed wrong**.

Scoring: The first problem you submit, if you get it right, will be worth 3 points, the second one 4 points, and so on until the last problem you submit is worth 8 points. Thus, each subround is worth **33 points** for a round total of **66 points**. We encourage you to devise a strategy for the round beforehand, and reconsider it after the first subround!

1. Alon uniformly selects two (not necessarily distinct) factors of 20^{22} . If the probability that at least one of these factors is divisible by the other can be expressed as a reduced common fraction $\frac{a}{b}$, compute the value of $a + b$.
2. Daniel has 2 stacks of sticks. In the first stack, there are 4 yellow sticks. In the second stack, there are 2 yellow sticks and 2 blue sticks.
They repeat the following process until one stack is empty: they randomly choose a stack at random with equal probability, then they uniformly at random choose a stick from that stack to throw away.
If the probability that the first stack is emptied while the second stack still contains both blue sticks can be expressed as a reduced common fraction $\frac{a}{b}$, evaluate $a + b$.
3. Hongning has 9 slips of paper labeled with the numbers 1 through 9. Reese has 8 slips of paper labeled with the numbers 1 through 8. Hongning and Reese each pick a slip uniformly at random from their respective piles. The probability that Reese's strip of paper has a larger number on it than Hongning's can be expressed as a reduced common fraction $\frac{a}{b}$. What is $a + b$?
4. Consider an arithmetic sequence of real numbers with common difference -3 . Let the sum of the first n terms be denoted by S_n . If $S_4 = 10$, compute S_7 .
5. The following 2-dimensional shape is created by removing a 1 by 2 rectangle from the middle third of a 3 by 3 square:



If the ratio of the volume of the figure created by rotating the shape 360 degrees around its left edge can be expressed in the form $a \cdot \pi$, what is a ?

6. Three vertices of a cube are selected uniformly at random, and a triangle is drawn connecting these selected vertices. If the probability that the resulting triangle is acute can be expressed as a reduced common fraction $\frac{a}{b}$, compute the value of $a + b$.