Time Limit: 15 minutes total, 2.5 minutes per submission.

Instructions: This subround consists of 6 questions, all of which have **integer answers**. You will be given all 6 questions at once (below), but you **must submit a previously unsubmitted problem every 2.5 minutes**. If a single problem is submitted multiple times, **only the first submission will be counted** and **all other submissions will be deemed wrong**.

Scoring: The first problem you submit, if you get it right, will be worth 3 points, the second one 4 points, and so on until the last problem you submit is worth 8 points. Thus, each subround is worth **33 points** for a round total of **66 points**. We encourage you to devise a strategy for the round beforehand, and reconsider it after the first subround!

1. Rithwick has a toilet paper roll in the shape of a hollow cylinder, where the hollowed part has a radius of 1 and the entire roll has a radius of 7. If Rithwick uses up half of the toilet paper roll, by volume, to achieve a new hollow cylinder still with inner radius 1, what is the new outer radius of his roll?

2. Define the *confused factorial* of n (denoted $n?!$) as the arithmetic mean of $(n-1)!$ and $(n+1)!$, where $n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$, and $0! = 1$. If the sum $\frac{1?!}{2!} + \frac{2?!}{3!} + \cdots + \frac{20?!}{21!}$ can be expressed as a reduced common fraction $\frac{a}{b}$, compute the value of $a+b$.

3. Alan has a square pyramid with volume V. If the base side lengths and the height are all increased by 50%, Alan obtains a new square pyramid with volume V'. If the ratio $\frac{V'}{V}$ can be expressed as a reduced common fraction $\frac{a}{b}$, compute $a+b$.

4. Owen selects an ordered quadruplet of (not necessarily distinct) positive integers (a, b, c, d) uniformly at random such that $a \cdot b \cdot c \cdot d = 336$. If the probability that at least one of a, b, c, and d is divisible by 14 can be expressed as a reduced common fraction $\frac{a}{b}$, compute the value of $a+b$.

5. There exist two distinct positive integers a and b, both between 1 and 100 (exclusive), such that 2022 leaves a remainder of 1 when divided by either a or b. Compute the value of $\left|a^2 - b^2\right|$.

6. Let $a_1, a_2, \ldots$ be an arithmetic sequence where $a_5 = 9$, $a_9 = 17$, and $2^{10} - 3$ is the n^{th} term of the sequence. Compute the value of n.