Time limit: 60 minutes.
Instructions: The Gallop round is a fast-paced, **live-scored** round that consists of 27 problems split into 9 sets of 3 questions each. Sets become progressively harder, but problems on later sets are worth more points (see below). The main rule for this round is that you **must** submit the answers to a set before moving on to the next set (you may put guesses if you don't know the answer, as incorrect answers are **not** penalized). You cannot return to a set once you've submitted your answers, so care must be taken to not move on too quickly.
Scoring: For each set, there is a specific number of points that will be awarded for correctly solving a problem in that set. The exact point values per problem are shown in the table below:

Round #	1	2	3	4	5	6	7	8	9	Total Pts.
Pts/Problem	10	11	12	13	15	17	19	22	25	432

This is set 1.

1. The expression $\frac{1-1+1-1+\cdots-1+1}{2}$, where the numerator contains 111 ones, can be written as a reduced common fraction $\frac{a}{b}$. What is $a+b$?

2. 7 people were writing problems for Mustang Math Tournament. The test needs 240 problems in total, 77 of which have already been written. If each person writes 3 problems per day, how many (whole-number) days will it take to reach 240 problems?

3. Max the Mustang is waiting impatiently to attend Mustang Spirit Week. He is watching a clock with a minute hand of length 6 units and a hour hand of length 3 units, and must wait until the minute hand has swept out an area of 12π square units. The area that the hour hand sweeps out in the same amount of time can be expressed as $\frac{a}{b}\pi$ square units, where $\frac{a}{b}$ is a reduced common fraction. What is $a+b$?

This is set 2.

4. Daniel is designing a rectangular jigsaw puzzle with 2300 square pieces, each of length 1 centimeter. What is the minimum perimeter, in centimeters, of the puzzle?

5. What is the smallest positive composite number that is not divisible by any of the 5 smallest positive primes?

6. There are two wooden slabs, each with 9 cut-out holes. The two wooden slabs are placed on top of each other such that every hole on the first slab overlaps with a unique hole on the second slab. Then, one hole from each slab is randomly covered up and a ball is randomly dropped from above one of the 9 original hole positions. If the probability that the ball will go through both slabs can be expressed as a reduced common fraction $\frac{a}{b}$, then what is $a + b$?

This is set 3.

7. When the three numbers $109, 331$, and 479 are divided by a positive integer N, they have the same remainders. What is the largest possible N?

8. A 6-digit number is called *rad* if it comes from writing a 3-digit number composed of 3 distinct digits twice in a row. For example, 123123 and 861861 are rad, but 122122 and 861816 are not. How many 6-digit rad numbers are divisible by 5?

9. How many quadratics of the form $f(x) = ax^2 + bx + c$ with positive integer coefficients are there such that $f(1) = 7$?

This is set 4.

10. Let r be the answer to question 12. Alice and Bob are running around two concentric circular tracks of radii $60r$ and $90r$ respectively. They both run at $3\,\text{units/s}$. Assuming Alice and Bob run forever, what is the furthest distance they will ever be from each other?

11. Let a be the answer to question 10. Compute

$$\sqrt[3]{a\sqrt[3]{a\sqrt[3]{a\cdots}}}.$$

12. Let n be the answer to question 11. In regular polygon $A_1 \ldots A_n$, compute the degree measure of angle $\angle A_1A_6A_n$.

This is set 5.

13. The following expression is written out on a whiteboard

$$2022^{2022^{2022^{\cdot^{\cdot^{\cdot^{2022}}}}}}$$

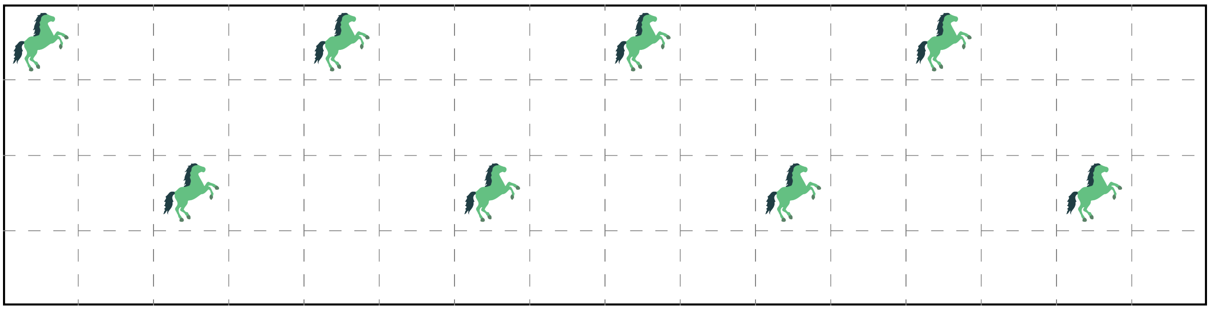
where 2022 is written 2022 times. Evan evaluates this expression from the top exponent to the bottom base (i.e., right to left), while Serena evalues this expression from the bottom base to the top exponent (i.e., left to right). Evan gets x as his result and Serena gets y as her result. What is the sum of the units digits of x and y?

14. A horse is standing on the circumference of a circle with radius 1. The horse spins around so that it faces a random direction. After that, the horse walks $\sqrt{3}$ units forward and stops. The probability that the horse ends up inside the circle can be expressed as a reduced common fraction $\frac{a}{b}$. What is $a + b$?

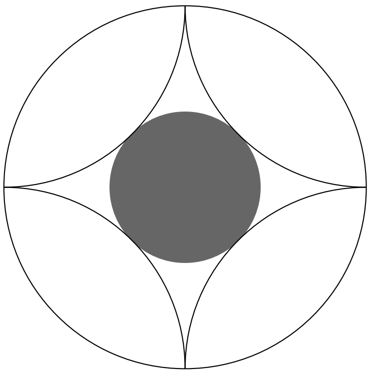
15. On each day n, Farmer Shak feeds his horse one carrot with n calories. His horse will continue eating these daily carrots until the total number of calories it has consumed is a multiple of 2022. How many days will Farmer Shak be able to feed his horse for?

This is set 6.

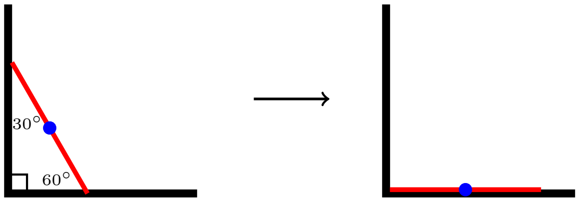
16. Given the following 4×16 barn, Noel wants to separate his 8 mustangs into 8 equally-sized **rectangular** stables so that no two mustangs are in the same stable and no empty spaces remain - the stables cover the entire barn. Provided that the mustangs are fixed in their shown positions, and he only divides them along the dotted lines, how many ways can he do this?



17. In the circle below, four quarter circle arcs are drawn with endpoints starting on the circumference of the outer circle. The shaded circle is externally tangent to all four of the quarter circle arcs. If the ratio of the radius of the inner shaded circle to the radius of outer circle can expressed as $\sqrt{a} - b$, where a and b are positive integers. Compute $a + b$.



18. Sebi places a 6-inch stick with a pen attached to its midpoint in a corner such that the stick forms a 30-60-90 triangle, as shown below. He then slides the stick while keeping both ends in contact with the walls until it lies flat against the wall it previously made a $60°$ angle with. If the length of the path (in inches) that the pen traces out can be expressed as $a\pi$, where a is a positive integer, then what is the value of a?



This is set 7.

19. A group of 2 boys and 2 girls are playing tag. A boy starts out as being "it." The probability that a girl is "it" after 6 random tags can be expressed as a reduced common fraction $\frac{a}{b}$. What is $a+b$?
(If you do not know, tag is a game where one person starts out as being "it." When they touch someone else, that person becomes "it" and the original person is no longer "it").

20. You are standing on the top right corner of a magical 2000 by 2022 grid of squares, all painted yellow. Then, you paint the square you're standing on green. Every second, you move one square to the left and one square down, and you paint the square you land on green (the magical grid teleports you to the rightmost column of squares if you cross the left edge, and it teleports you to the top row if you cross the bottom edge). Eventually, you get back to the same square you started at. How many squares have been painted green at that time?

21. If x and y are 2 positive integers chosen uniformly at random, then the probability that x^y and y^x have the same remainder when divided by 3 can be expressed as a reduced common fraction $\frac{a}{b}$. What is $a+b$?

This is set 8.

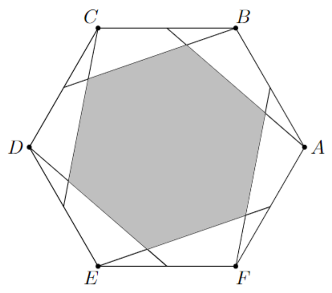
22. Let a, b be positive integers that satisfy

$$2a\sqrt{3b} + 3b\sqrt{2a} - \sqrt{4044a} - \sqrt{6066b} + \sqrt{12132ab} = 2022.$$

Compute the sum of all possible values of a.

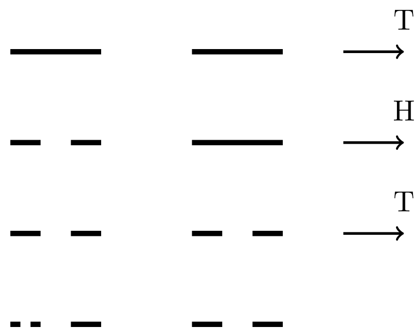
23. A sequence is constructed so that any given term of the sequence (except the first) is the perimeter of an equilateral triangle whose area is the preceding term. If the first three terms form a geometric sequence, and the value of the 3rd term can be expressed as $a\sqrt{b}$, find $a+b$

24. Consider regular hexagon $ABCDEF$. We form another regular hexagon by connecting each vertex of $ABCDEF$ to the midpoint of the side adjacent to it, as shown in the diagram. If the ratio of the area of the smaller hexagon to the area of $ABCDEF$ can be expressed as the reduced common fraction $\frac{a}{b}$, then what is $a+b$?



This is set 9.

25. Contor is playing with a rope of length L. He begins by cutting out the middle third. Then, every minute, he flips a coin. If it lands tails he cuts out the middle third of the leftmost segment of rope and if it lands heads he cuts out the middle third of the rightmost segment of rope. For example, the diagram below shows the result of the rope after the sequence of flips THT. The expected number of minutes until the length of the rope is less than or equal to $\frac{11L}{27}$ can be expressed as a reduced common fraction $\frac{a}{b}$. What is $a+b$?



26. Consider a $4m$ by n grid, where m and n are positive integers chosen uniformly at random from the range $[1, 100]$, inclusive. Define the "Manhattan Distance" between two cells in a grid to be the sum of their horizontal distance and vertical distance (the cells $(1, 3)$ and $(2, 1)$ have a Manhattan distance of $|2-1|+|1-3| = 3$). Owen wants to use the colors blue and red to color some cells of the grid with the following restrictions:

    - For any two distinct cells colored the same color, the Manhattan Distance between them must be even.
    - For any two distinct cells colored different colors, the Manhattan Distance between them must be odd.
    - It is ok for **none** of the cells to be colored or **all** of the cells to be colored.
    - No cell can be colored with both colors.

    Suppose $f(m, n)$ denotes the number of such possible colorings. Compute the expected value of k, where k is the largest integer such that $f(m, n)+1$ is divisible by 2^k.

27. Given a regular octagonal prism, we define a *diagonal* as any line segment connecting two distinct vertices of the prism. How many pairs of diagonals in a regular octagonal prism intersect (not counting intersections at a vertex of the prism)?