Time limit: 60 minutes.

Instructions: This test contains 24 short answer questions to be solved in 60 minutes. Each column will contain problems of a specific subject area: Algebra, Number Theory, Mixed-Subjects, Geometry, and Combinatorics. The specific subject area is written above their respective column in the bingo board below. Note that the problems are not ordered by difficulty in any way. **No calculators.**

Scoring: One point will be awarded for each correctly solved problem. Additionally, two points will be awarded for each "bingo", defined as correctly solving **all** of the problems in a row, column, or long diagonal (bottom left to top right or top left to bottom right) of the following grid. Note that the center square of the grid is a free space that is considered to be correct when evaluating bingos. In total, there are 24 points available from solving problems and 24 points available from the bingos (5 rows, 5 columns, 2 long diagonals) for a maximum score of 48 points.

Algebra	NT	Mixed	Geo	Combo
1	6	11	15	20
2	7	12	16	21
3	8	Free	17	22
4	9	13	18	23
5	10	14	19	24

1. Bill can drive to his favorite beach resort on either a 60-mile-long northern route or a 40-mile-long southern route. If Bill drives on the northern route, he travels on average 20 miles per hour faster and reaches the resort in half the time it takes him if he drives on the southern route. What is Bill's average speed in miles per hour on the northern route?

2. What is the base-10 value of

$$121_{2022} - 121_{2021} + 121_{2020} - 121_{2019} + \cdots + 121_{24} - 121_{23}$$

where subscripts denote bases?

3. A geometric sequence with n^{th} term a_n satisfies $a_3 = a_4 \cdot a_5 \cdot a_6 = 8$. What is $a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \cdot a_6 \cdot a_7 \cdot a_8 \cdot a_9$?

4. Andrew and David are having a contest to see who can make a bigger number. At $t = 0$ minutes, Andrew starts with 512 and David starts with 9. In every subsequent minute $(t = 1, 2, \ldots)$, Andrew squares his current number and David cubes his current number. After how many minutes will David have a greater number than Andrew for the first time?

5. Let the two roots of $x^2 - 9x + 16$ be the leg lengths of a right triangle. What is the perimeter of the triangle?

6. What is the number of digits in $20^{22} \cdot 5^{28}$?

7. Let E and O be the sum of the even and odd factors of 210, respectively. Compute $E - O$.

8. What is the remainder when $19 \cdot 20 \cdot 22 \cdot 23$ is divided by 21?

9. What is the only three-digit prime number whose digits multiply to 30?

10. What positive integer x satisfies $\text{lcm}(20, x) + \text{lcm}(22, x) = 528$?

11. Kaity constructs a 3-letter string by selecting letters uniformly at random from $\{M, A, T, H\}$ with replacement. If the probability that his string is a rearrangement of MMT can be expressed as a reduced common fraction $\frac{a}{b}$, compute $a + b$. Note that MMT also counts as a rearrangement.

12. Picachu the modern Picasso is plotting points on a coordinate plane. He first draws $A_1 = (12, 23)$ and $A_2 = (10, 25)$. For $n \geq 3$, he draws A_n such that A_{n-1} is the midpoint of $\overline{A_n A_{n-2}}$. Given that X is the sum of the x-coordinates of $A_1, A_2, A_3, \ldots, A_{20}$ and Y is the sum of the y-coordinates of $A_1, A_2, A_3, \ldots, A_{20}$, compute $X + Y$.

13. How many Pythagorean triples of positive integers (a, b, c), where $1 \leq a, b, c \leq 100$ and $a^2 + b^2 = c^2$, satisfy $c - b = 1$?

14. The square with vertices $(0, 0)$, $(6, 0)$, $(6, 6)$, and $(0, 6)$ is drawn on the coordinate plane. Justin constructs a convex quadrilateral by selecting a non-corner lattice point uniformly at random from each side of the square as its vertices. If the probability that the area formed by the quadrilateral is exactly half the area of the square can be expressed as a reduced common fraction $\frac{a}{b}$, compute $a + b$.

15. Let D be on $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$. If $AB = 5$, $AC = 9$, and $BC = 10$, $CD - BD$ can be expressed as a reduced common fraction $\frac{a}{b}$. Compute $a + b$.

16. Let O_1, O_2 be two circles of radius 8 with each circle passing through the other's center. Let their intersections be A and B. Let O_3 be a circle with center A and radius $\overline{AB}$. If O_3 intersects O_1 and O_2 at distinct points C and D which are not B, compute CD.

17. Let $ABCD$ be a square with area 144. Let points E, F, G, and H be on line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ respectively such that $AE = BF = CG = DH = 4$. Let M be the intersection between $\overline{EG}$ and $\overline{FH}$. What is the area of $AEMH$?

18. In regular hexagon $ABCDEF$, a random point P is chosen in its interior. If the probability that the area of quadrilateral $ABCP$ covers at least half of the area of $ABCDEF$ can be expressed as a reduced common fraction $\frac{a}{b}$, compute $a + b$.

19. Axel gave his students Yuuki and Tristan identical right hexagonal prisms of clay, with all the edges being 3 inches. He then told them to cut the clay in half. While Tristan cut it in half by cutting into opposite edges (center diagram), Yuuki cut it in half by cutting into opposite vertices (right diagram). Each of their cuts made a hexagonal cross-section across their clay. If the ratio between the areas of Tristan's cross-section to Yuuki's cross-section is R, R^2 can be expressed as a reduced common fraction $\frac{a}{b}$. Compute $a + b$.
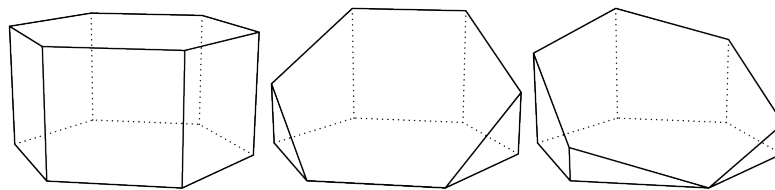
20. In the cutest auto battler, every pet has a health stat and an attack stat. Feeding various food items changes the pet's stats as shown in the table below. If a hummingbird starts out with 2 Attack and 2 HP, what is the minimum number of food items that Justin must feed it such that it reaches exactly 50 Attack and 50 HP? The pet must have a non-negative health and attack stat at all times.

Food	Attack	HP
Peach	+0	+2
Fried Shrimp	+5	−3
Broccoli	−1	+3

21. Arpit the Ant is looking for a path to go back to MMT HQ. If Arpit only travels on the grid lines drawn below, may only move left, right, and upwards, and never goes back to a point where he's already been, how many paths are there for Arpit to travel?

MMT HQ

Arpit the Ant
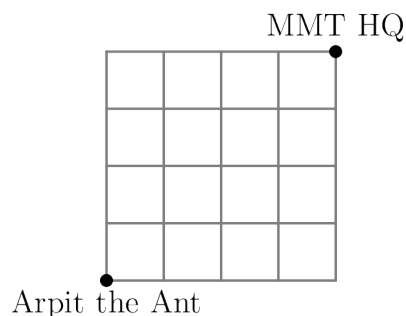
22. Grace rolls a fair 6-sided die twice. If the probability that on the first roll, Grace gets a 3 or lower, but the sum of the two die rolls is greater than 6 can be expressed as a reduced common fraction $\frac{a}{b}$, compute $a + b$.

23. Yash the STONK master is buying and selling stocks. Each day, he has a 50% chance to make 300 dollars and 50% chance to lose 100 dollars. If the probability that Yash does not have a net loss after 8 days can be expressed as a reduced common fraction $\frac{a}{b}$, compute $a + b$.

24. Aileen is making a fruit bowl to gift Anna for Mother's Day. Aileen tosses in an odd number of oddly-shaped apples, some number of pairs of pears (an even number of pears), and some number of bananas that come in bunches of 5. Anna counts that there are 200 pieces of fruit in the bowl. However, because Anna just pulled an all-nighter, her count might be off by up to 2. Given that Aileen tossed in at least one of each fruit, how many different combinations of apples, pears, and bananas may be in the bowl?