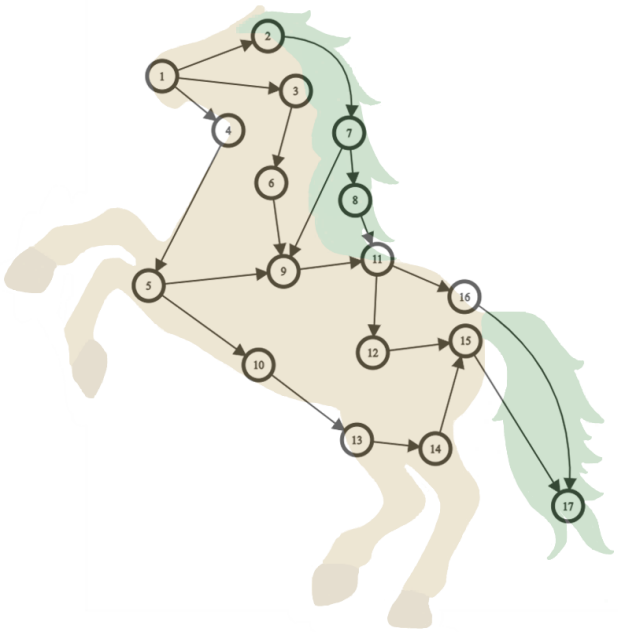
RELAY RODEO ROUND RULES

[Relay Rodeo Answer Submission Form]

1. The "Relay Rodeo" Round will consist of 17 questions to be solved in 60 minutes.
2. Like traditional relay rounds, questions utilize the answers from previous questions to arrive at a new answer. Unlike traditional relay rounds, this relay is done as a TEAM - all team members may see all questions at once rather than a single individual getting a subset of the questions.
3. Additionally, this round isn't strictly linear - some questions may have multiple output answers that split off into multiple paths, and others may take in multiple input answers and combine them into a single path! This ensures there are multiple ways to complete the relay even if your team can't solve the problems within a single path.
4. ***Below is the path tree for the 2021 Relay Rodeo.*** Teams start from question 1 and work their way down to 17, each arrow representing an output and input from their respective questions.



5. Each problem is worth n+1 points, where n is the number of problems you have to solve before it to be able to solve the current one. For example, to solve problem 11, you need the answers to each of 1 through 9, so problem 11 is worth 9+1=10 points. Here is a table of the point values:

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	**Total**
Points	*1*	*2*	*2*	*2*	*3*	*3*	*3*	*4*	*8*	*4*	*10*	*11*	*5*	*6*	*15*	*11*	*17*	***107***

RELAY RODEO ROUND

This round works the following way: Each question will have one or more inputs (except the first question), an answer submission that should be filled into the answer submission form, and one or more outputs (except the last question) to be used in future problems that may or may not be the same as the answer submission. The inputs for a question will be denoted in the form '(*#n*).' If you see this in a question, you should replace it with the input from question number n. The final answer which should be submitted for a question will be clearly denoted at the end of each question, and the outputs of the question will appear in italicized text below the problem.

1. This question serves as an easy sample question to show you how things work. Let $x = 3$, $y = 3$, $z = 4$. Your answer submission for this question is $x + y + z$. ***(1 point)***
   Your output for #2 is x.
   Your output for #3 is y.
   Your output for #4 is z.

2. Let T be your input from (*#1*). 5^{T+2} people get on an empty train. At the first stop, T^{T+1} people get off, and $T + 1$ people get on. At the second stop, $(T + 5)^T$ people get off and nobody gets on. Your answer submission for this question is the number of people on the bus after the second stop. ***(2 points)***
   Your output for #7 is the number of people on the bus after the second stop (same as your answer submission).

3. I have a water bottle that is in the shape of a cylinder with a cone on top. The radius of the cylinder and cone are both 4. The height of the cone is $4 \cdot$(*#1*). I fill my water bottle up with some amount of water such that when the bottle's circular face is on the table, the water fills up to $\frac{1}{44}$ of the total height, and when the bottle is held upside down (so the cone is pointing straight down), the water fills up to $\frac{3}{11}$ of the total height. Your answer submission for this question is the height of the cylindrical portion of the bottle. ***(2 points)***
   Your output for #6 is the height of the cylindrical portion of the bottle (same as your answer submission).

4. $2\cdot$(*#1*) people are standing in a line to buy tickets to the Relay Rodeo, of which 3 are part of the same family and 2 other people are friends. Your answer submission is the number of different orders the people can be in if the family has to be together (stand in consecutive positions), and friend A has to be somewhere before friend B in line. ***(2 points)***
   Your output for #5 is the number of different orders the people can be in if the family has to be together, and friend A has to be somewhere before friend B in line (same as your answer submission).

5. There are (*#4*) students at RHS. Due to COVID-19, at least half of them have to stay home. You are told that the number that stays home is divisible by 7, leaves a remainder of 4 when divided by 11, and leaves a remainder of 6 when divided by 17. Your answer submission for this question is the positive difference between the number of students that go back to school and the number that stay home. ***(3 points)***
   Your output for #9 is the number of students that stay home.
   Your output for #10 is the number of students that go to school.

6. On the planet Mustang, a stallion can be traded for 6 colts, a colt can be traded for 2 foals, and (*#3*) foals can be traded for a stallion. You cannot do trades in reverse (i.e. 6 colts cannot be traded for a stallion) and trading 2 stallions for 12 colts counts as 2 trades (since you do it twice). Arpit visits this planet with 1 stallion. Your answer submission is the minimum number of trades Arpit needs to do to have at least 1000 stallions. ***(3 points)***
   Your output for #9 is the minimum number of trades Arpit needs to do to have at least 1000 stallions (same as your answer submission).

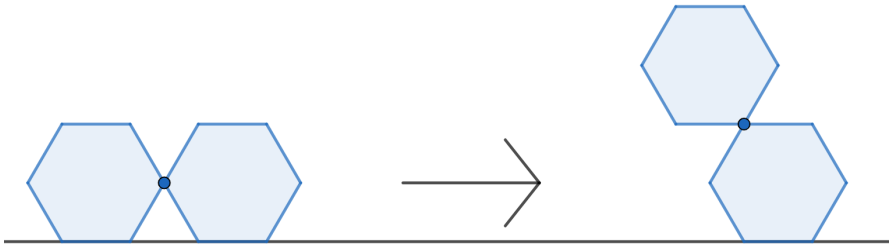
7. Find 3 palindromes that add up to (*#2*) such that the difference between the largest and smallest of the 3 palindromes is minimized. Your answer submission is the difference between the largest and smallest palindromes you found. ***(3 points)***
   Your output for #8 is the largest of the three palindromes.
   Your output for #9 is the smallest of the three palindromes.

8. Your answer submission is the exponent of 2 in the prime factorization of the number of ways to write (*#7*) as a sum of [sum of the prime factors of (*#7*)] positive integers. ***(4 points)***
   Your output for #11 is the exponent of 2 in the prime factorization of the number of ways to do this (same as your answer submission).

9. You are in a room with [the greatest prime factor of (*#6*)] other people, playing a game where each person picks an integer between 1 and (*#5*), inclusive. Whoever chooses the integer such that the sum of the positive differences with the other numbers is greatest wins (*#7*) dollars. The other people choose their integers uniformly at random, but you are allowed to pick any integer that you want. Everyone chooses their numbers at the same time. If there is a tie, the winners each win (*#7*) dollars. Your submission is the average amount of money you will win, rounded down, assuming you play optimally. ***(8 points)***
Your output for #11 is the average amount of money you win, rounded down (same as your answer submission).

10. A flower is made by attaching [the smallest prime factor of (*#5*)] circular petals of radius 1 meter to its stem of negligible radius, such that the petals are parallel to the ground and the petals may overla p. If the maximum surface area of petals that is exposed to the sun (the area visible when observing from a top-down view) can be expressed in simplest form as $a\pi + \frac{b\sqrt{c}}{d}$, your answer submission for this question is $a+b+c+d$. ***(4 points)***
Your output for #13 is the value of a+b+c+d (same as your answer submission).

11. [(*#8*)-1] regular hexagons with side length one are each placed on top an infinite line, with the distance between the center of one hexagon to the center of the next being [(*#9*)$+1$] units. The leftmost hexagon starts rolling to the right. When a hexagon meets another they fuse together to form a new object, and stay attached by the vertex they meet at, continuing to roll together as depicted in the diagram below. We define a 'roll' to be whenever a new vertex of a rolling object (any number of hexagons connected together) touches the ground, and a 'revolution' to be any time an object makes a full 360° rotation about the object's center. Your answer submission for this question is the sum of the number of complete rolls and complete revolutions the object makes by the time all the hexagons meet. ***(10 points)***
The output to #12 is the number of complete rolls the object makes.
The output to #16 is the number of complete revolutions the object makes.



12. Murray has a weighted die that has a $\frac{1}{3}$ chance each of rolling a 1 or a 3, and $\frac{1}{12}$ chance each of rolling a 2, 4, 5, or a 6. He rolls the die (*#11*) times. Your answer submission is the expected value of the sum of the die's rolls. ***(11 points)***
The output to #15 is the expected value of the sum of the die's rolls (same as your answer submission).

13. Your answer submission is the number of ways to complete the Relay Rodeo problems up to and including number (*#10*) such that you complete the prerequisites of a problem before solving the problem. ***(5 points)***
The output to #14 is the number of ways to do this (same as your answer submission).

14. A (*#13*)-sided die has the numbers 1 through (*#13*) printed on it, with no leading zeros. Your answer submission is the total number of digits on the die. ***(6 points)***
The output to #15 is the total number of digits on the die (same as your answer submission).

15. You are given a function $f(x) = Ax^3 + Bx^2 + Cx + D$. We know $f(0) =$(*#12*), $f(-1) =$(*#14*), and $f(2) = 5177$. Your answer submission is the value of $9A + 3B + 3C - D$. ***(15 points)***
The output to #17 is the value of 9A+3B+3C-D (same as your answer submission).

16. $10\cdot$(*#11*) can be written as $a_1 \cdot 1^1 + a_2 \cdot 2^2 + a_3 \cdot 3^3 + \cdots + a_n \cdot n^n$ for nonnegative integers a_1 through a_n. Your answer submission is the minimum value of $a_1 + a_2 + \cdots + a_n$. ***(11 points)***
The output to #17 is the minimum value of the sum of the coefficients (same as your answer submission).

17. Murray and Marty have both been given two mystery numbers in different bases. Murray was given the number $0.\overline{1}$ in base (*#15*) and Marty was given the number $0.\overline{1}$ in base (*#16)*, with both numbers having infinitely repeating ones. If the positive difference between Murray and Marty's values can be expressed in simplest form as $\frac{a}{b}$, your answer submission is $a + b$. ***(17 points)***