MOUNTING MAYHEM ROUND RULES

Mounting Mayhem Answer Submission Form

1. The Mounting Mayhem Round will consist of 17 mathematical-based puzzle questions. Students will have 60 minutes to solve as many of them as possible.
2. Puzzles can range from logic to number theory to geometry and more. They often require out-of-the-box thinking although all topics can readily be understood and grasped by middle schoolers.
3. The maximum possible score is 50 points. Harder puzzles will be worth more points, but these puzzles will be longer and/or require more creativity — you must decide what problems to focus your time on to optimize your team score.

MOUNTING MAYHEM ROUND

1. Create 25 using 5 0's and any mathematical operations. Your answer may include parentheses. ***(1 point)***

2. Claire loves numbers. In particular, she loves numbers which she considers pretty. You were able to obtain a set of numbers which you have been told are pretty:

$$5,\ 21,\ 87,\ 341,\ 1365,\ 5461$$

Unfortunately, one of these numbers is not actually pretty! Claire wants you to find this number based on some algorithmic formula. Which number is not pretty? ***(1 point)***

3. The four symbols below each represent a different integer value. Given the numbers on the right represent the sum of each row and the numbers on the bottom represent the sum of each column, determine the sum of the second column. ***(1 point)***

				31
				46
				52
				47
48	?	40	50	

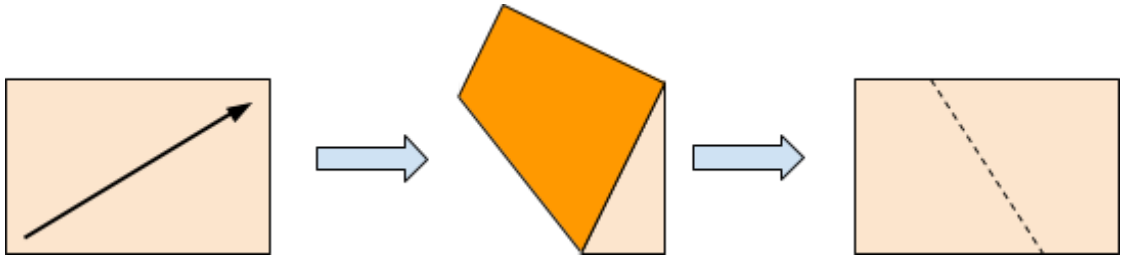
4. Jimmy invents a game called Let's Not Make 24. In this game, you need to find the number of ways to not make 24 given a set of 3 numbers, using each number from the set exactly once. The game has three rules:

   1) only the standard $+$, $-$, $/$, and $\times$ operations are allowed, although operations may be reused,
   2) parentheses are not allowed
   3) different orderings of numbers are unique*. How many ways can one Not Make 24 given the following set of three numbers: 3, 3, and 7? ***(2 points)***

* Order matters e.g. $3+3+7$ and $3+7+3$ are different ways to not make 24.

5. Noah's calculator is malfunctioning. The only buttons that work are "5", "11", "$+$", and "$=$". As a result, Noah can print some numbers but not others. For example, Noah can print 27 by typing "$5 + 11 + 11 =$", but he can't print 17. What is the largest number that Noah cannot print on his calculator? ***(2 points)***.

6. It's origami time! Given a rectangular paper with sides of length of 5 and 10, the length of the crease from overlaying one corner with the opposite, diagonal corner can be written in the simplified radical form: $\frac{a\sqrt{b}}{c}$. Find $a \cdot b \cdot c$. ***(2 points)***



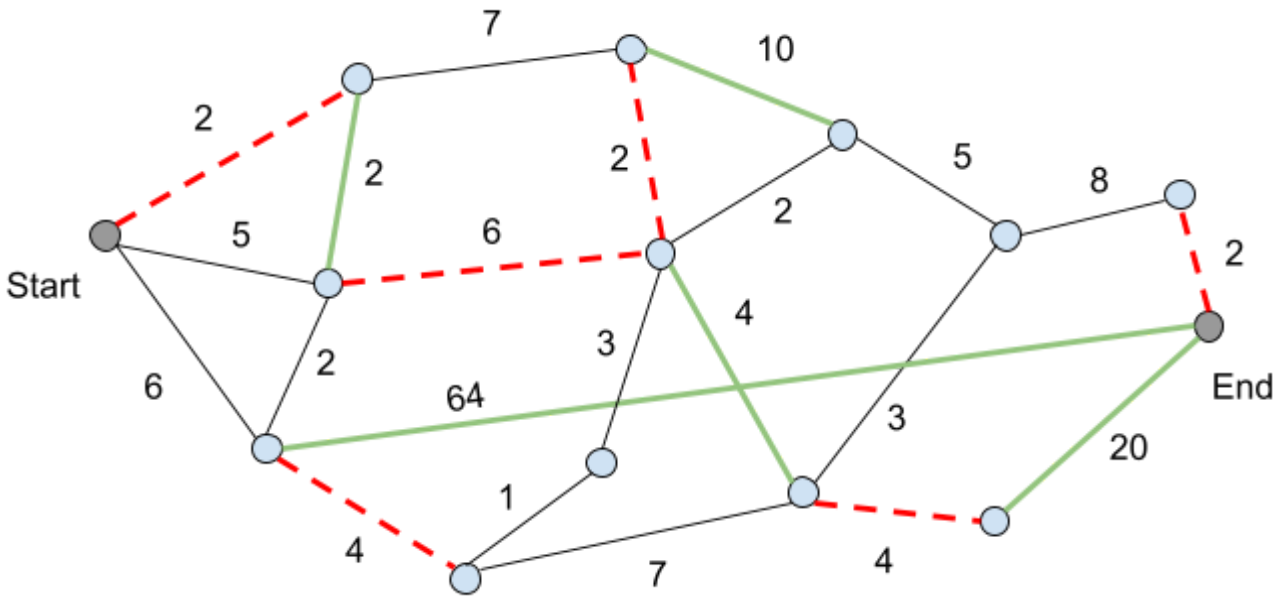
7. Grace and Wilson are playing a game. Grace begins by making a 4-digit number code containing digits from 1 through 5 (with repeats allowed). The goal of the game is for Wilson to keep guessing a new four-digit number until he cracks Grace's four-digit code. However, in between guesses, Wilson gets valuable information. If a digit in his guess appears in Grace's number and in the correct placement (units, tens, hundreds, etc.), Grace gives him a white pin, indicated by a W. If a digit in his guess appears in Grace's number albeit in the wrong placement, Grace gives him a black pin, indicated by a B.* Given four of Wilson's guesses are shown below, what is Grace's four-digit code? ***(3 points)***

1) 1123 WBB
2) 2345 BB
3) 1253 WWB
4) 4531 BB

Example: WBB means Wilson's numerical guess shares 3 digits with Grace's code (1 digit in correct placement, 2 digits in incorrect placement).

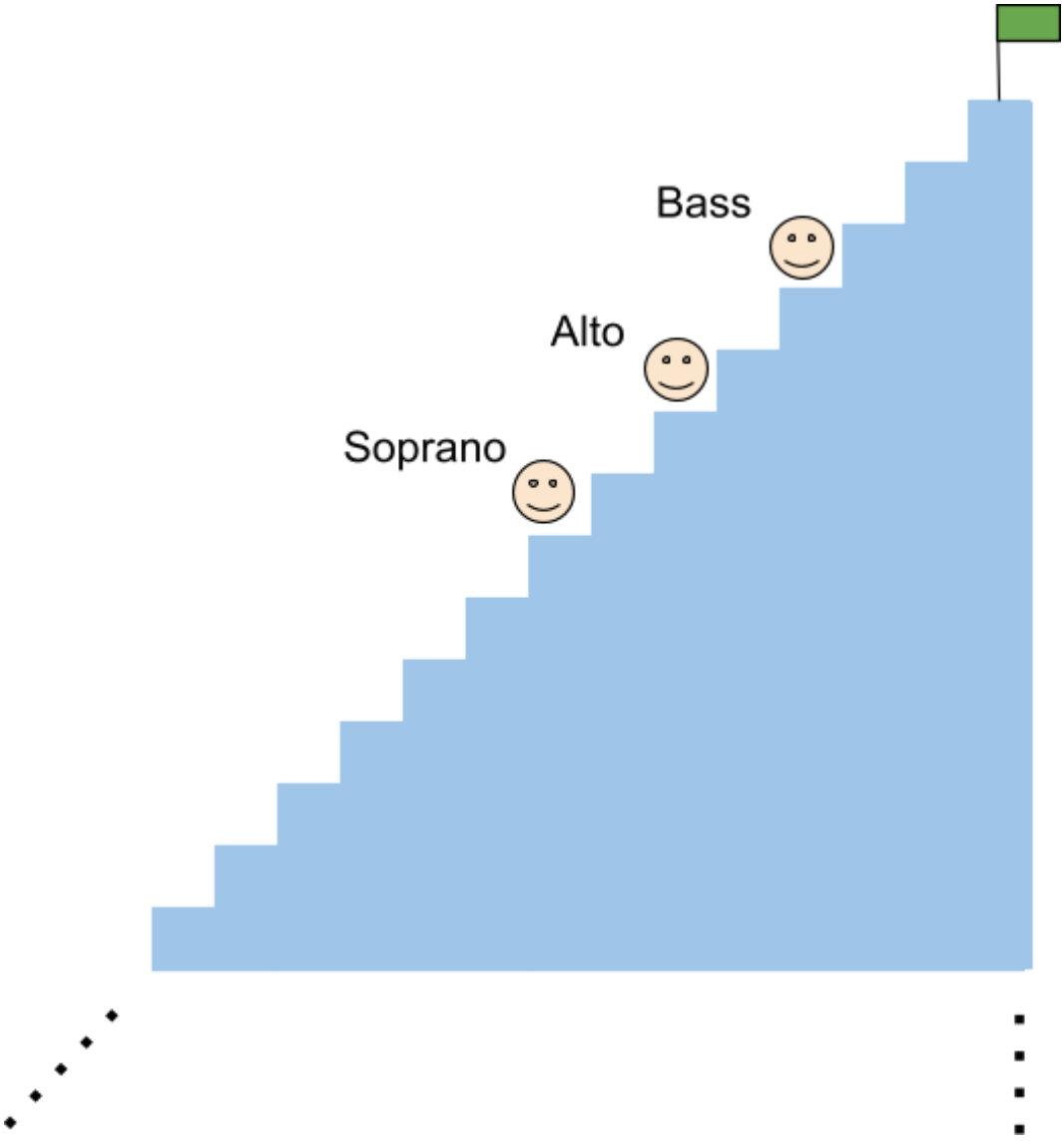
* If a digit repeats multiple times in Wilson's guess, Wilson will only get additional pins if Grace also repeats that same digit multiple times in her code. Moreover, the order of pins is not considered important in the game.

8. Oh no! The Mounting Mayhem event is starting soon and you're running late! Use this map to find the fastest way to get there! Roads marked in red are traffic-heavy lanes and take twice as long as the indicated time. Roads marked in green are highways and take half as long as the indicated time. For example, a normal lane marked 4 takes 4 minutes to traverse, a green lane marked 4 takes 2 minutes to traverse, and a red lane marked 4 takes 8 minutes to traverse. Determine the shortest time (in minutes) it will take for you to reach Mounting Mayhem! ***(3 points)***



9. Find the next number in the sequence: $5, 4, 10, 12, 25, 32, 65, 84, 170 \ldots$ ***(3 points)***

10. Bass, Alto, and Soprano are all in a staircase climbing race. Bass climbs at 3 steps/second, Alto climbs at 4 steps/second, and Soprano climbs at 5 steps/second. Before the race, they all stand at the foot of the staircase (step 0, if you will). Moreover, to make up for their different speeds, Bass gets an integer head start (in seconds) over Alto, and Alto gets a different integer head start (in seconds) over Soprano. A photographer takes a picture near the end of the race an integer number of seconds after the start of the race. Based on the photo, what is the minimum possible number of steps in the race? ***(3 points)***



11. Given that all five following sub-questions have different *answer choices*, submit the **answers** to sub-questions 1 through 5 as a string of 5 consecutive letters (e.g. ABCDE). ***(3 points)***

Note: All *answer choices* are in lowercase, and all **answers** are in UPPERCASE

1) This sub-question's *answer choice* is the same letter as the **answer** to sub-question 5
a) **B**
b) **C**
c) **D**
d) **E**
e) **A**

2) Hmm.. this sub-question's **answer** happens to be alphabetically adjacent to the *answer choice* of the sub-question above it!
a) **C**
b) **D**
c) **B**
d) **A**
e) **E**

3) Assume A $= 1$, B $= 2$, C $= 3 \ldots$ Then, sum the other sub-questions' **answers**, add 3, then take the remainder when divided by 5. Convert back to a letter, and I am your *answer choice*!
a) **C**
b) **D**
c) **A**
d) **E**
e) **B**

4) Contrary to popular belief, the **answer** to this sub-question is not a vowel:
a) **B**
b) **E**
c) **A**
d) **C**
e) **D**

5) Contrary to popular belief, the **answer** to this sub-question is unknown:
a) **C**
b) **D**
c) **B**
d) **A**
e) **E**

12. A magician has a set of 10 cards labeled 1 through 10 and arranges them randomly into two, equal sets (set A and set B) with 5 cards each. He first takes the highest card in set A and the lowest card in set B and records their absolute difference. He then takes the 2nd highest card in set A and the 2nd lowest card in set B and records their absolute difference. He continues this process until he has 5 absolute differences, then he sums all these differences. What is the probability that the magician's final sum is equal to 25? ***(3 points)***

13. A pyramid can be thought of as a lattice of points with the apex representing a single point, a 2×2 grid of points a meter below, a 3×3 grid of points 2 meters below, etc. all the way to the base 99 meters below: a 100×100 grid of lattice points (refer to image below). Arpit descends the pyramid by randomly selecting one of the four closest lattice points below him, moving to it, and again selecting one of the 4 lattice points below him, and so on. Let X be the probability that Arpit's route ends in an interior lattice point of the base (i.e. not on the edge) expressed as a simplified common fraction. Then, the numerator of X can be written as the sum or difference of three unique powers of 2. Find the sum of exponents of all powers of 2 in the numerator of X. ***(4 points)***

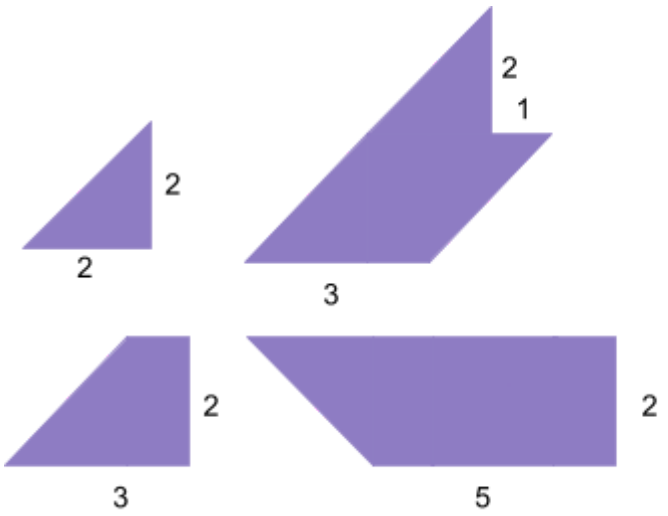
A cross-section of the top 4 meters of the pyramid
Note: Arpit's route can travel through the interior lattice points in the pyramid

14. A number is labeled a doppelganger if any digit from 1 to 9 appears an even number of times within a number (e.g. 1231320 is a doppelganger). A number is labeled the Nth all-inclusive number if it's the concatenation of all integers from 1 to N (e.g. 1234567891011 is the 11th all-inclusive number). Find the sum of all N from 1 to 100 for which the Nth all-inclusive number is a doppelganger. ***(4 points)***

15. Find the number represented by **MOUNTS** in the summation below if each letter represents a unique digit from 0 to 9 and all instances of the same letter represent the same digit. ***(5 points)***

$$\begin{array}{r} \texttt{GROUND} \\ \texttt{+MOUNT} \\ \hline \texttt{MOUNTS} \end{array}$$

16. Arrange the following 4 shapes to make a symmetrical, two-dimensional letter T. All pieces are to-scale and all angles are multiples of $45°$. Let M be the sum of the perimeters of all four shapes. Let N be the perimeter of the closed figure T. Find $(M-N)^2$. ***(5 points)***



"This is what the T should look like, albeit a different scale."

17. Alice and Bob are playing a game. First, Alice says a random integer from 2 to 10. Then, Bob takes Alice's integer and either

    1) multiplies it by an integer between 2 to 9 inclusive or
    2) adds it to a integer between 2 to 9 inclusive

    Alice then takes Bob's integer and

    1) either multiplies it by a integer between 2 to 9 inclusive or
    2) adds it to a integer between 2 to 9 inclusive

    This pattern repeats until one person says a integer strictly greater than 1000. The first person to say a integer greater than 1000 wins. For example, one play through might look like so:

    Alice says 2. Bob then multiplies it by 7, and says 14. Alice then adds 2, and says 16. Bob then multiplies it by 5, and says 80. Alice then multiplies it by 2, and says 160. Bob then adds 8 and says 168. Alice then multiplies it by 7, and says 1176. The game ends, and Alice wins.

    If Alice and Bob both play optimally, what is the sum of all integers that Alice can say at the start (from 2 to 10) and still guarantee a win? ***(5 points)***